Regular Language and Logic

mardi 10 novembre 2015 13:32

Simulating logic formulas

Let $\varphi(x_1, \dots, x_m)$ be a formula on an automatic structure $\langle E, r_1, \dots, r_k \rangle$ where the only free variables the x_i (first order variables)

We design an ω-automaton $\mathcal{A}$ such that $\mathcal{L}(\mathcal{A})$ is $\mathcal{J}(\varphi)$:
$$\mathcal{J}(\varphi) = \{ (u_0^{(0)}, \dots, u_0^{(m-1)}) \dots (u_k^{(0)} \dots u_k^{(m-1)}) \dots \mid u^{(0)} = \mathcal{J}(e_0), \ \vdots \ , u^{(m-1)} = \mathcal{J}(e_{m-1}), \ (e_0, \dots, e_{m-1}) \text{ is true} \}$$

Firstly, one changes φ:

- remove all functions turning them into predicate
- remove all universal quantifications $(\forall)$ replacing them by a $\neg \exists \neg$
- change all "$\exists v$" by "$\exists v \ v \in E \wedge$"
- replace all implications $A \to B$ by $\neg A \vee B$

Secondly, one build the ω-automaton recursively:

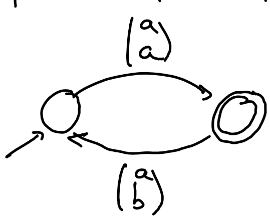
- If φ is an atomic formula (hence a predicate) it corresponds to an ω-automaton

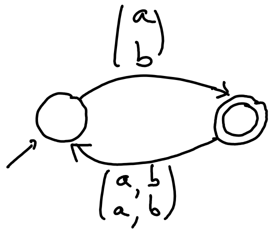
- If φ is $\varphi_1 \vee \varphi_2$ [resp $\varphi_1 \wedge \varphi_2$] construct the automaton which recognize the "union" [resp. "intersection"] of the automata for φ_1 and φ_2

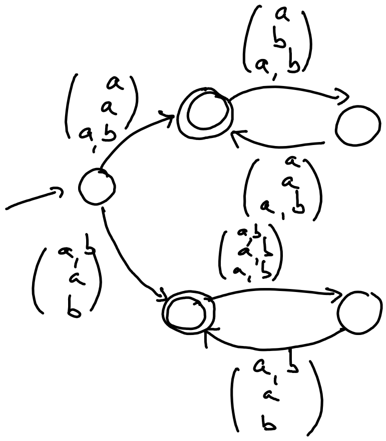
⚠ beware alphabets: $\varphi_1(x,y) \vee \varphi_2(y,z)$

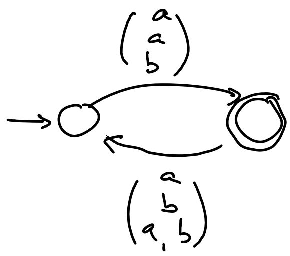
$\swarrow$ alphabet $= \Sigma^2$ $\qquad$ $\searrow$ alphabet $= \Sigma^2$

but for φ, alphabet $= \Sigma^3$

Ex φ_1: 

; φ_2 

$\varphi_1 \vee \varphi_2$: 

; $\varphi_1 \wedge \varphi_2$ 

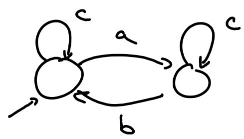
- If φ is $\neg\psi$, construct the ω-automaton which recognizes the complement language

↳ firstly turn the ω-automaton into a Muller deterministic ω-automaton (Safrah algorithm) ← the costly procedure

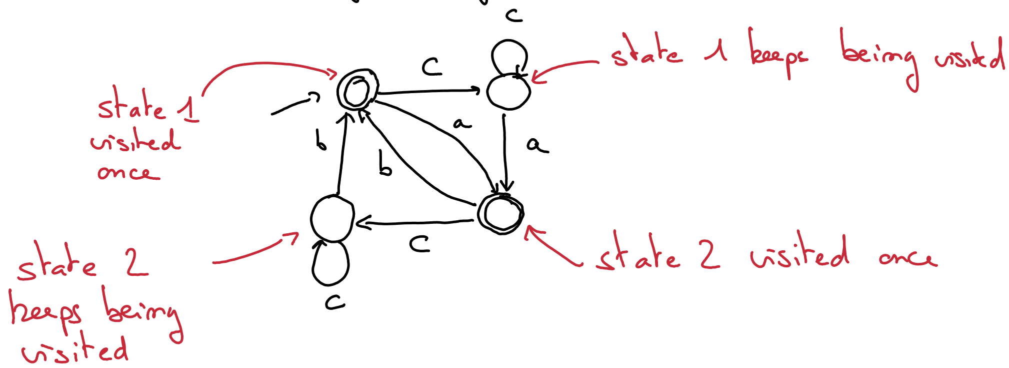
↳ secondly, reverse the set of accepting set of states of states

↳ then, turn it back to an Büchi ω-automaton

[guess the set of states which are visited infinitely many times and check it with the Büchi condition

ex:  and both states

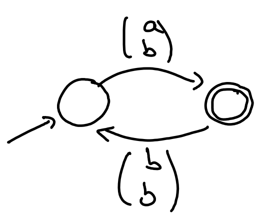
infinitely many times visited :



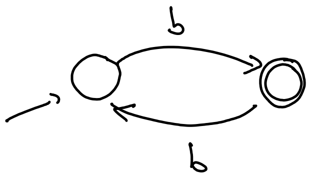
$\Rightarrow$ ensure cycle state 1 visited ⇄ state 2 visited

- If φ is $\exists x\ \varphi(x)$, erase the letter corresponding to x in the alphabet

Ex $\varphi(x, y)$:



$\exists x\ \varphi(x, y)$



For close formulas, alphabet is unary the formula is true if the language is not empty

Ex: $E = \mathbb{N}$ $\mathcal{J}$ = "lsb first binary representation"

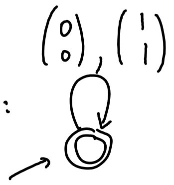
$\sigma = \langle +, = \rangle$

$\varphi = \forall x \ \exists y \ \exists z \quad x + y = z$

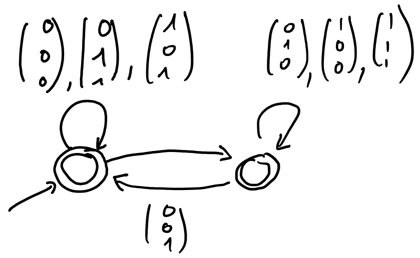
Transforming φ: $\neg \exists x, x \in \mathbb{N} \wedge \neg \exists y \ y \in \mathbb{N}$

$\wedge \ Plus(x, y, t) \wedge t = z$

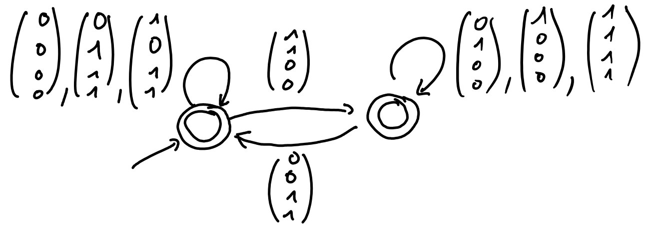
$t = z$: $\begin{pmatrix} z \\ t \end{pmatrix}$



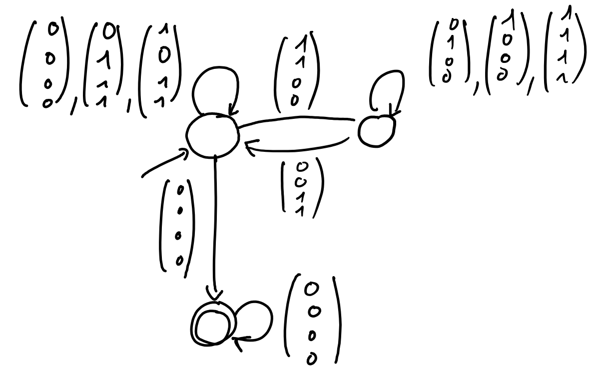
$Plus(x, y, t)$: $\begin{pmatrix} x \\ y \\ t \end{pmatrix}$



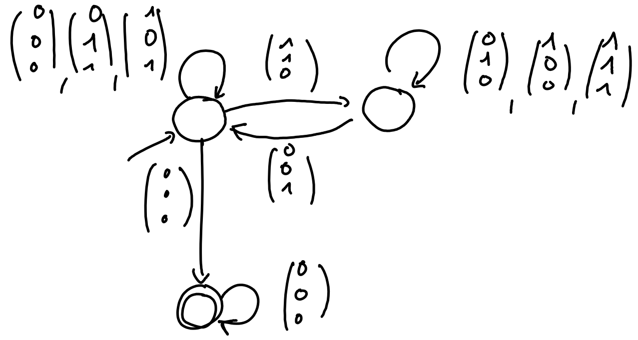
$t = z \wedge Plus(x, y, t)$: $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$



$t \in \mathbb{N} \wedge t = z \wedge Plus(x, y, t)$: $\begin{pmatrix} x \\ y \\ z \\ t \end{pmatrix}$

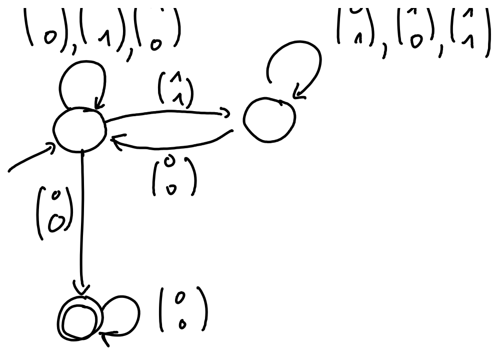


$\exists t, t \in \mathbb{N} \wedge t = z \wedge \ldots$: $\begin{pmatrix} x \\ y \\ z \end{pmatrix}$



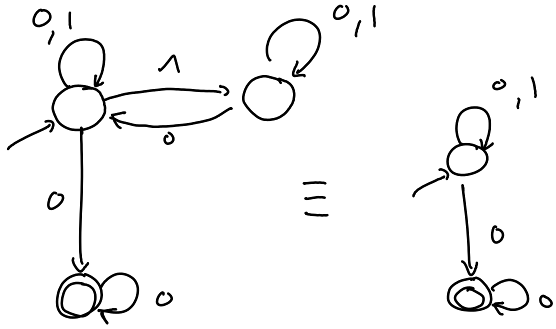
$\exists z\ z \in \mathbb{N} \wedge \exists r, t \in \mathbb{N} \wedge t = z \wedge \ldots$:

$\begin{pmatrix} x \\ y \end{pmatrix}$



$\exists y\ y \in \mathbb{N} \wedge \exists z\ z \in \mathbb{N} \wedge \exists r \ldots$:

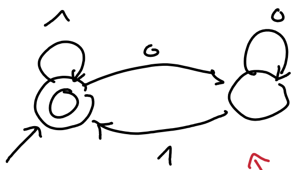
(x)



finitely many 1

$\neg \exists y\ y \in \mathbb{N} \wedge \exists z\ z \in \mathbb{N} \ldots$:

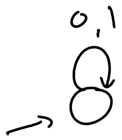
(x)



infinitely many 1

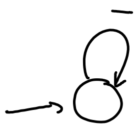
$x \in \mathbb{N} \wedge \neg \exists y \ldots$:

(x)

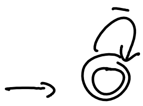


empty

$\exists x,\ x \in \mathbb{N} \wedge \ldots$:



$\neg \exists x, \ldots$:



full the formula is true

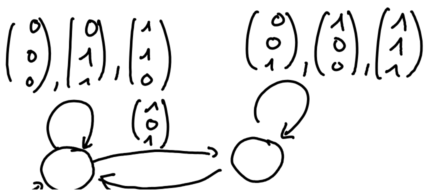
Ex2: $\varphi = \forall x\ \forall y\ \exists z\quad x + z = y$

Transforming φ: $\neg \exists x\ x \in \mathbb{N} \wedge \exists y\ y \in \mathbb{N} \wedge \neg \exists z\ z \in \mathbb{N} \wedge \exists t\ t \in \mathbb{N}$

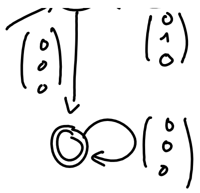
$\wedge\ Plus(x, z, t) \wedge y = t$

$\exists t\ t \in \mathbb{N} \wedge Plus(x, z, t) \wedge y = t$

$/ x \backslash$

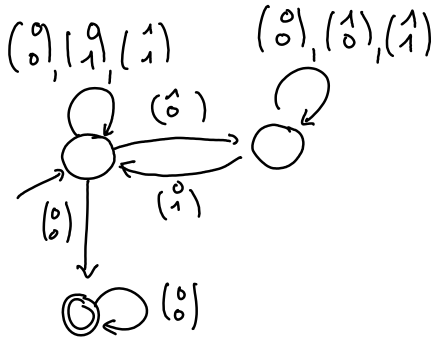


$\begin{pmatrix} y \\ z \end{pmatrix}$



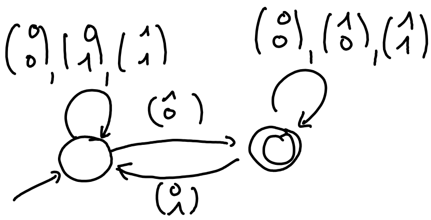
$\exists z \; z \in \mathbb{N} \wedge \exists t \; t \in \mathbb{N} \wedge Plus(x,z,t) \wedge y = t$

$\begin{pmatrix} x \\ y \end{pmatrix}$ basically $x \leq y$



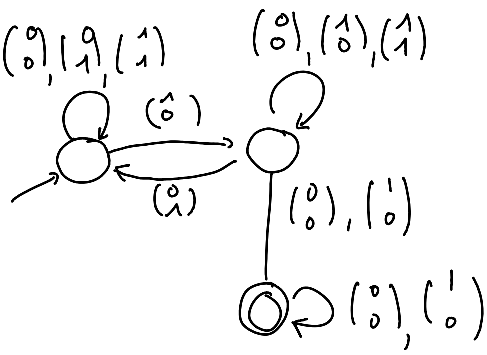
$\neg \exists z \; z \in \mathbb{N} \wedge \exists t \; t \in \mathbb{N} \wedge \ldots$

$\begin{pmatrix} x \\ y \end{pmatrix}$



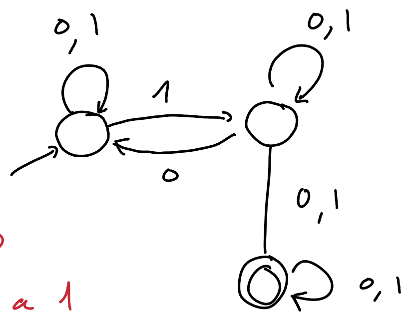
$y \in \mathbb{N} \wedge \neg \exists z \; z \in \mathbb{N} \wedge \ldots$

$\begin{pmatrix} x \\ y \end{pmatrix}$



$\exists y \; y \in \mathbb{N} \wedge \neg \exists z \; z \in \mathbb{N} \wedge \ldots$

(x)

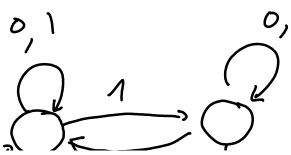


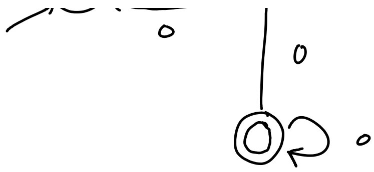
words with a 1

↳ only 0 is such that $\forall y \; \exists z \quad 0 + z = y$

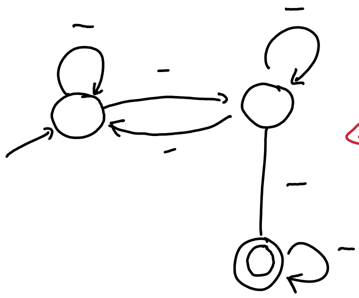
$x \in \mathbb{N} \wedge \exists y \; y \in \mathbb{N} \wedge \ldots$

(x)



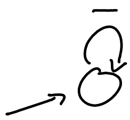


$\exists x, x \in \mathbb{N} \wedge \exists y \ldots$



full

$\varphi = \neg \exists x, x \in \mathbb{N} \wedge \ldots$



empty

The formula is false; the previous automata could give counter examples ($x \neq 0$): this is called synthesis